Code No: 123AH **R15**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, February - 2024

MATHEMATICS – III

(Common to EEE, ECE, EIE)

Time: 3 hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART - A

(25 Marks)

1.a) Define Cauchy's linear differential equation. [2]
b) What are the ordinary point and regular singular points of a differential equation? [3]
c) State Legendre's equation. [2]
d) Prove that $J_0^1 = -J_1$. [3]
e) Define an analytic function and give an example. [2]
f) State Cauchy's generalized integral formula. What is the use of it? [3]
g) State the Maclaurin's series expansion of f(z) in a complex plane. [2]
h) Define a pole of order n. Give a suitable example. [3]
i) What is the condition for the existence of conformal mapping? [2]
j) Discuss the transformation w= z + c, where c is a complex constant. [3]

PART - B

(50 Marks)

2. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin\left[2\log(1+x)\right]$ [10]

OR

3. Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0$. [10]

4. Prove that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x)$. [10]

OR

5. State and prove orthogonal property of Legendre's polynomials. [10]

6. Prove that the function f(z) defined by $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, z \neq 0 \text{ and } f(0) = 0$ is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f^1(0)$ does not exists. [10]

OR

7. Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ around $C: |z-1| = 3.$ [10]

8. Find the Laurent's series expansion of $\frac{7z-2}{z(z+1)(z-2)}$ in the annulus $1 < |z+1| < 3$.

[10]

OR

9. Find the residue of $f(z) = \frac{z^3}{(z-1)^4 (z-2)(z-3)}$ at its poles and hence evaluate $\oint_C f(z) dz$ where C is the circle $|z| = \frac{1}{2}$. [10]

10. Find the image of the rectangle $-\frac{\pi}{2} \le x \le \frac{\pi}{2}, -1 \le y \le 1$ under the mapping $w = \sin z$.

[10]

OR

11. Find the bilinear transformation which maps the points $z = 1, i, -1$ on to the points $w = i, 0, -i$. Hence find (a) the image of $|z| < 1$ and (b) the invariant points of the transformation. [10]

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